# ALGEBRA - Part 1

Quick Study® ACADEMIC

properties.dividing.equations.factoring

## SET THEORY

### NOTATION

- { } braces indicate the beginning and end of a set notation; when listed, elements or members must be separated by commas. **EX: A = {4, 8, 16}**; sets are finite (ending, or having a last element) unless otherwise indicated.
- ... indicates continuation of a pattern. **EX: B = {5, 10, 15, ... , 85, 90}**
- ... at the end indicates an infinite set, that is, a set with no last element. **EX: C = {3, 6, 9, 12, ...}**
- | is a symbol which literally means "such that."
- ∈ means "is a member of" OR "is an element of." **EX:** If **A = {4, 8, 12}** then **12 ∈ A** because **12** is in set **A**.
- ∉ means "is not a member of" OR "is not an element of." **EX:** If **B = {2, 4, 6, 8}** then **3∉B** because **3** is not in set **B**.
- ∅ means empty set OR null set; a set containing no elements or members, but which is a subset of all sets; also written as { }.
- ⊂ means "is a subset of;" also may be written as ⊆.
- ⊄ means "is *not* a subset of;" also may be written as ⊈.
- **A⊂ B** indicates that every element of set **A** is also an element of set **B**. **EX:** If **A = {3, 6}** and **B = {1, 3, 5, 6, 7, 9}** then **A⊂B** because the **3** and **6** which are in set **A** are also in set **B**.
- $2^n$ is the number of subsets of a set when **n** equals the number of elements in that set. **EX:** If **A = {4, 5, 6}** then set **A** has **8** subsets because **A** has **3** elements and $2^3 = 8$.

### OPERATIONS

- ∪ means union.
- **A∪ B** indicates the union of set **A** with set **B**; every element of this set is either an element of set **A** OR an element of set **B**; that is, to form the union of two sets, put all of the elements of both sets together into one set making sure not to write any element more than once. **EX:** If **A = {2,4}** and **B = {4, 8, 16}** then **A ∪ B = {2, 4, 8, 16}**.
  - ∩ means intersection.
  - **A∩B** indicates the intersection of set **A** with set **B**; every element of this set is also an element of **BOTH** set **A** and set **B**; that is, to form the intersection of two sets, list only those elements which are found in **BOTH** of the two sets. **EX:** If **A = {2,4}** and **B = {4, 8, 16}** then **A ∩ B = {4}**.
- $\overline{A}$ indicates the complement of set **A**; that is, all elements in the universal set which are **NOT** in set **A**. **EX:** If the Universal set is the set of Integers and **A = {0, 1, 2, 3, ...}** then $\overline{A}$ **{-1, -2, -3, -4,...}**. A ∩ $\overline{A}$ = ∅.

### PROPERTIES

- **A = B** means all of the elements in set **A** are also in set **B** and all elements in set **B** are also in set **A**, although they do not have to be in the same order. **EX:** If **A = {5, 10} and B = {10, 5}** then **A = B**.
- **n(A)** indicates the number of elements in set **A**. **EX:** If **A = {2, 4, 6}** then **n(A) = 3**.
- ~ means "is equivalent to"; that is, set **A** and set **B** have the same number of elements although the elements themselves may or may not be the same. **EX:** If **A = {2, 4, 6}** and **B = {6, 12, 18}** then **A ~ B** because **n(A) = 3** and **n(B) = 3**.
- **A ∩ B = ∅** indicates disjoint sets which have no elements in common.

### SETS OF NUMBERS

- **Natural or Counting numbers** = **{1, 2, 3, 4, 5, ..., 11, 12, ...}**
- **Whole numbers** = **{0, 1, 2, 3,..., 10, 11, 12, 13, ...}**
- **Integers** = **{... , -4, -3, -2, -1, 0, 1, 2, 3, 4, ...}**
- **Rational numbers** = **{p/q | p** and **q** are integers, **q ≠ 0}**; the sets of Natural numbers, Whole numbers, and Integers, as well as numbers which can be written as proper or improper fractions, are all subsets of the set of Rational numbers.
- **Irrational numbers** = **{ x | x** is a Real number but is not a Rational number}; the sets of Rational numbers and Irrational numbers have no elements in common and are therefore disjoint sets.
  - **Real numbers** = **{x | x** is the coordinate of a point on a number line}; the union of the set of Rational numbers with the set of Irrational numbers equals the set of Real numbers.
  - **Imaginary numbers** = **{*ai* | a** is a Real number and ***i*** is the number whose square is **-1**}; $i^2 = -1$; the sets of Real numbers and Imaginary numbers have no elements in common and are therefore disjoint sets.

## PROPERTIES OF REAL NUMBERS

**FOR ANY REAL NUMBERS a, b, AND c**

| PROPERTY | FOR ADDITION | FOR MULTIPLICATION |
|---|---|---|
| **Closure** | **a + b** is a Real number | **ab** is a Real number |
| **Commutative** | **a + b = b + a** | **ab = ba** |
| **Associative** | **(a + b) + c = a + (b + c)** | **(ab)c = a(bc)** |
| **Identity** | **0 + a = a** and **a + 0 = a** | **a • 1 = a** and **1 • a = a** |
| **Inverse** | **a + (-a) = 0** and **(-a) + a = 0** | **a •** $^1/_a$ **= 1** and $^1/_a$ **• a = 1** if **a ≠ 0** |
| **Distributive Property a(b + c) = ab + ac; a(b - c) = ab - ac** | | |

### PROPERTIES OF EQUALITY

**FOR ANY REAL NUMBERS a, b, AND c**

**Reflexive:** a = a
**Symmetric:** If **a = b** then **b = a**
**Transitive:** If **a = b** and **b = c** then **a = c**
**Addition Property:** If **a = b** then **a + c = b + c**
**Multiplication Property:** If **a = b** then **ac = bc**
**Multiplication Property of Zero:** **a • 0 = 0** and **0 • a = 0**
**Double Negative Property:** **-(-a) = a**

### PROPERTIES OF INEQUALITY

**FOR ANY REAL NUMBERS a, b, AND c**

**Trichotomy:** Either **a > b**, or **a = b, or a < b**
**Transitive:** If **a < b**, and **b < c** then **a < c**
**Addition Property of Inequalities:** If **a < b** then **a + c < b + c**
If **a > b** then **a + c > b + c**
**Multiplication Property of Inequalities:** If **c ≠ 0** and **c > 0,** and **a > b** then **ac > bc;**
also, if **a < b** then **ac < bc**
If **c ≠ 0** and **c < 0,** and **a > b** then **ac < bc;**
also, if **a < b** then **ac > bc**

## OPERATIONS OF REAL NUMBERS

### ABSOLUTE VALUE

**|x| = x** if **x** is zero or a positive number; **|x| = -x** if **x** is a negative number; that is, the distance (which is always positive) of a number from zero on the number line is the absolute value of that number. **EXs: | - 4| = - (- 4) = 4; |29| = 29; |0 | = 0; | - 43| = - (- 43) = 43**

### ADDITION

If the **signs** of the numbers are the **same**: **add** the absolute values of the numbers; the sign of the answer is the same as the signs of the original two numbers. **EXs: -11 + -5 = -16** and **16 + 10 = 26**

If the **signs** of the numbers are **different**: **subtract** the absolute values of the numbers; the answer has the same sign as the number with the larger absolute value. **EXs**: **-16 + 4 = -12** and **-3 + 10 = 7**

### SUBTRACTION

**a - b = a + (-b); subtraction is changed to addition of the opposite number**: that is, change the sign of the second number and follow the rules of addition (never change the sign of the first number since it is the number in back of the subtraction sign which is being subtracted; **14 - 6 ≠ - 14 + - 6**).
**EXs: 15 - 42 = 15 + (-42) = -27; - 24 - 5 = - 24 + (-5) = - 29; - 13 - (- 45) = -13 + (+45) = 32; - 62 - (-20) = - 62 + (+20) = - 42**

### MULTIPLICATION

The product of two numbers which have the **same** signs is **positive**; **EXs: (55)(3) = 165; (- 30)(- 4) = 120; (- 5)(- 12) = 60**

The product of two numbers which have **different** signs is **negative** no matter which number is larger. **EXs: (- 3)(70) = - 210; (21)(- 40) = - 840; (50)(-3) = - 150**

### DIVI...

***(DIVISORS ... ZE...)***

The quotient of two numbers ... he **same sign is posi...**
**EXs**: **(- 14) / (-7) = 2; (44) / (11)** ...
The quotient of two numbers wh... matter which number is larger.
**EXs**: **(-24) / (6) = -4; (40) / (-8)** ... **; (-14) / (56)** ...

### DOUBLE NEGATIVE

**- (- a) = a**; that is, the negative sign changes the sign of the contents of the parentheses. **EXs: - (-4) = 4; - (-17) = 17**